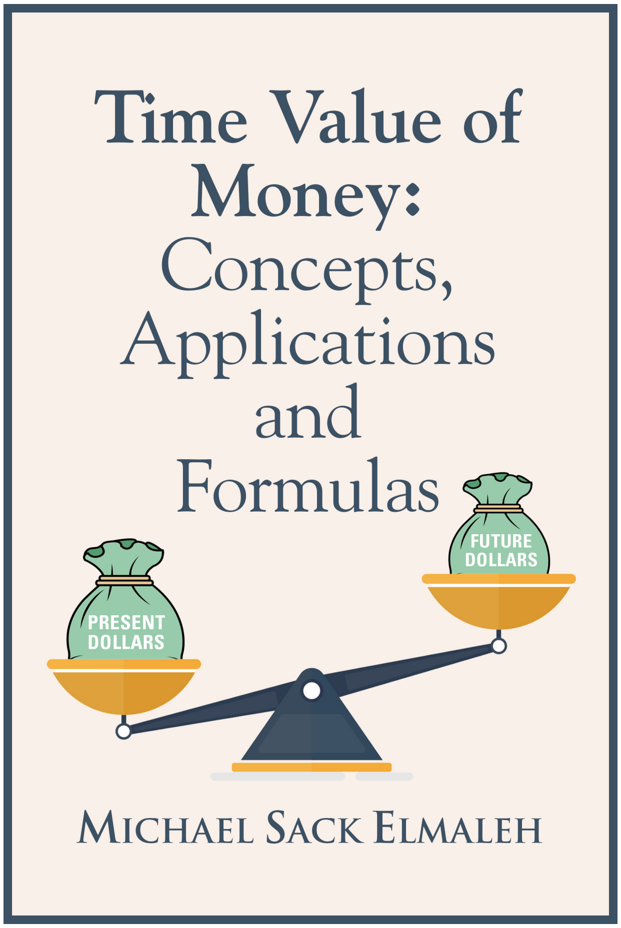
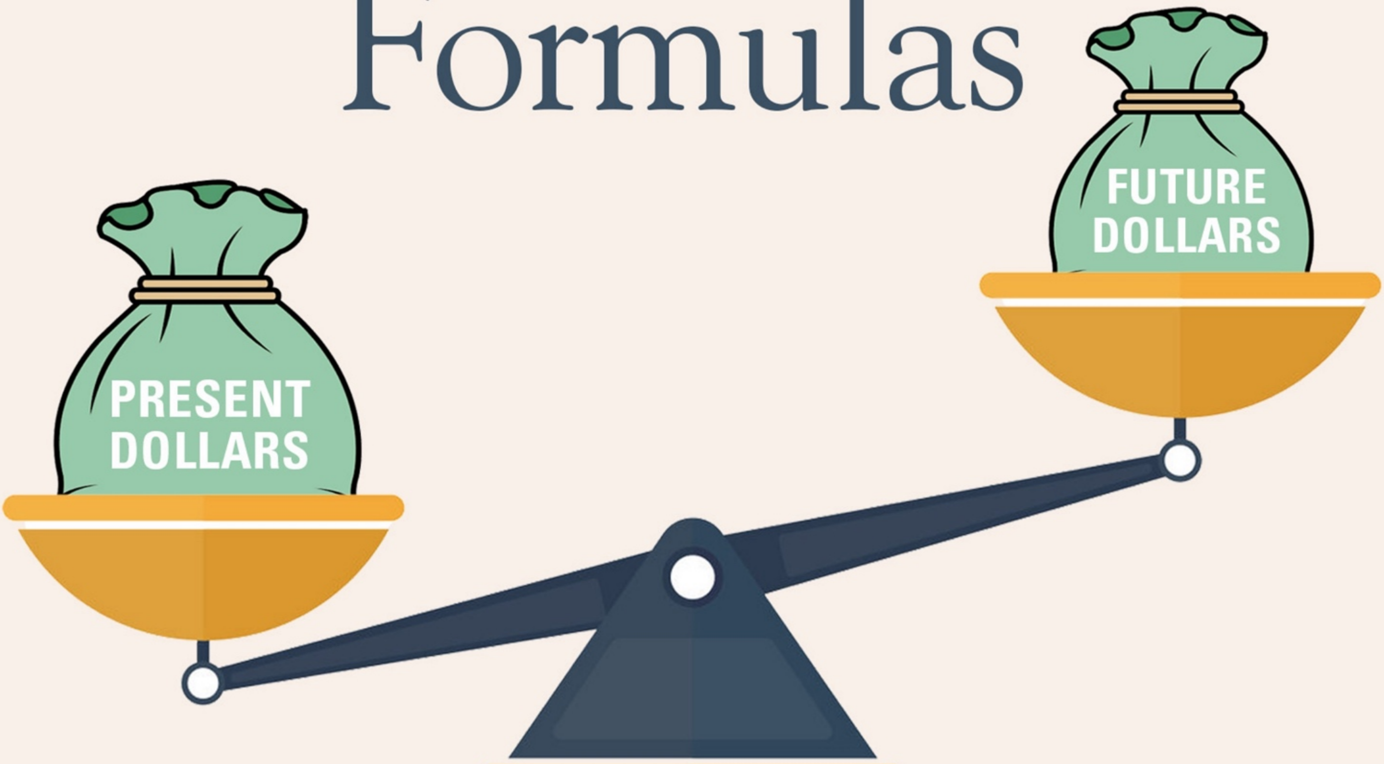


A complete easy to understand guide to time value of money concepts, applications, and formulas. The book covers compound interest, opportunity costs, discount rates, interest rates, present and future values.

Time Value of Money: Concepts, Applications and Formulas

By Michael Sack Elmaleh

Time Value of Money: Concepts, Applications and Formulas



Michael Sack Elmaleh

Time Value of Money:

Concepts, Applications and Formulas

Michael Sack Elmaleh CPA, CVA

Ebook ISBN: 978-1-64719-763-6

Published by BookLocker.com, Inc., St. Petersburg, Florida.

Printed on acid-free paper.

BookLocker.com, Inc.
2021

DISCLAIMER

This book details the author's personal experiences with and opinions about time value of money concepts, applications and formulas. The author is not a licensed financial consultant.

The author and publisher are providing this book and its contents on an "as is" basis and make no representations or warranties of any kind with respect to this book or its contents. The author and publisher disclaim all such representations and warranties, including for example warranties of merchantability and financial advice for a particular purpose. In addition, the author and publisher do not represent or warrant that the information accessible via this book is accurate, complete or current.

The statements made about products and services have not been evaluated by the U.S. government. Please consult with your own Certified Public Accountant or financial services professional regarding the suggestions and recommendations made in this book.

Except as specifically stated in this book, neither the author or publisher, nor any authors, contributors, or other representatives will be liable for damages arising out of or in connection with the use of this book. This is a comprehensive limitation of liability that applies to all damages of any kind, including (without limitation) compensatory; direct, indirect or consequential damages; loss of data, income or profit; loss of or damage to property and claims of third parties.

You understand that this book is not intended as a substitute for consultation with a licensed financial professional. Before you begin any financial program, or change your lifestyle in any way, you will consult a licensed financial professional to ensure that you are doing what's best for your financial condition.

This book provides content related to topics of finances and economic living. As such, use of this book implies your acceptance of this disclaimer.

About the Author

Michael Sack Elmaleh, Certified Public Accountant and Certified Valuation Analyst, is the author of *Enterprise Goodwill in Small Service Businesses: Negotiating a Fair Price* and *Financial Accounting: A Mercifully Brief Introduction*. Elmaleh has graduate degrees in accounting and philosophy and an undergraduate degree in psychology. He has served as an adjunct instructor at universities and colleges in Wisconsin and Maryland teaching accounting, mathematics, statistics and economics. He has lectured for continuing legal education courses on business valuation topics. He has published several peer reviewed articles on valuation theory and serves on the editorial board of *The Journal of Business Valuation and Economic Loss Analysis*. He has testified as an expert witness in Wisconsin and Maryland courts. He operates a practice specializing in the appraisal of closely held firms in Miami.

Table of Contents

Chapter 2: Compound Interest

Let's begin with interest calculations. Let's suppose you open a savings account, and you deposit $5,000 today. We call this $5,000 the principal or present value (PV). The ***compounding rate*** or ***period*** defines how frequently the bank will credit your account for interest earned. In the formulas that follow, and on most calculators, this compounding period is represented by the letter "***n*".**

The interest rate per compounding period is usually abbreviated by the letter "***i*".** Sometimes the interest rate is represented by the "%" symbol. Usually, you have to enter the interest rate per compounding period. So, for example, if your annual interest rate is 5% and you have four compounding periods per year you would enter 1.25% into your calculator.

There are two ways in which a bank might pay or credit you interest. One way they could pay you interest is simply to compute it based on your original $5,000 deposit. This is called ***simple interest***. In the case of a 5% annual rate, you'd earn $250 a year. If you didn't add to the principal, each year you'd get $250 added to your account. The following year, the interest would still be calculated only on the original principal.

The more typical way banks credit interest is the ***compound interest*** method. In the compound method, interest is credited based on the original principal ***plus*** the previously credited interest. You earn interest on the previously earned interest as well as the original principal. This compounding of course assumes that you have not withdrawn the previously credited interest earned.

The table below illustrates both simple and compound interest computations assuming an interest rate of 5% compounded annually. So, starting with $5,000 today we would accumulate $6,381 by the end of year five under the compounding method versus $6,250 or about $131 less using the simple interest approach.

Simple vs. Compound Interest				
i = 5%, n =5, PV= $5,000				
End of year	Simple Interest Credited	Account Balance	Compound Interest Credited	Account Balance
1	$250	$5,250	$250	$5,250
2	$250	$5,500	$263	$5,513
3	$250	$5,750	$276	$5,788
4	$250	$6,000	$289	$6,078
5	$250	$6,250	$304	$6,381

Table 2.1

Almost all computations of interest involving financial instruments and accounts utilize compounding computations. So, it is important to note the difference between what is called ***nominal*** versus ***effective interest*** rates. Usually, a bank will advertise both a nominal interest rate and an effective interest rate. A nominal rate is usually a rate assuming one annual compounding period. But most banks and other financial institutions will compound interest more frequently. The relationship between the nominal and effective interest rate is described by this formula[1]:

Effective Interest Rate Formula
$i_e = [1+(i_a/n)]^n-1$
where,
i_e = effective interest rate
i_a = nominal or annual interest rate
n = number of compounding periods in year

Table 2.2

This next table shows the relationship between the nominal and effective interest rate holding the number of compounding periods constant at 12. You can see that the effective interest increases with increasing nominal interest rate.

Impact of Nominal Interest Rate on Effective Interest Rate			
Nominal Interest Rate	Compounding Periods	Effective Interest Rate	% Difference
1.00%	12	1.0046%	0.46%
2.00%	12	2.0184%	0.92%
3.00%	12	3.0416%	1.39%
4.00%	12	4.0742%	1.85%
5.00%	12	5.1162%	2.32%
6.00%	12	6.1678%	2.80%
7.00%	12	7.2290%	3.27%
8.00%	12	8.3000%	3.75%
9.00%	12	9.3807%	4.23%
10.00%	12	10.4713%	4.71%
11.00%	12	11.5719%	5.20%
12.00%	12	12.6825%	5.69%

Table 2.3

A very interesting pattern emerges when a nominal interest rate is held constant and we greatly increase the number of compounding periods, not just within one year but over several years. In this next table we assume $1 is put in a bank account and will earn 5% nominal interest over 20 years. In the last column you begin to see the accumulated balance get closer and closer to a very important number known as "***e***", or Euler's constant[2]. As the compounding periods increase the accumulated balance will get closer and closer to 2.71828 which is ***e*** to six significant digits. This constant appears in many important applications, particularly in probability and statistics.

Increasing Total Compounding Periods and Euler's Constant				
Nominal Interest Rate	Compounding Periods/Per Year	Effective Interest Rate	Total Compounding Periods $20 \times n$	Accumulated Principal $(1+i)^{20n}$
5%	1	5.00%	20	2.653297705
5%	2	5.06%	40	2.685063838
5%	4	5.09%	80	2.701484941
5%	12	5.12%	240	2.712640285
5%	24	5.12%	480	2.715455682
5%	52	5.12%	1040	2.716976113
5%	365	5.13%	7300	2.718095668

Table 2.4

Finally, the last column of this table contains the general compound interest formula for computing the future value of a lump sum. This lump sum we have been calling the present value or initial deposit. The accumulated balance we will refer to as the future value.

Compound Interest Formula

$FV = PV \times (i + 1)^n$

FV = Future Value Accumulated

PV = Present Value

i = interest return per compounding period

n = number of compounding periods

Table 2.5

Again, the above formula and example assumes that all interest is reinvested or compounded.

Importantly, the above formula allows us to convert one lump sum payment in the future to an equivalent present value and vice versa. By rearranging terms, we arrive at the formula for converting a future value into a present value:

Lump Sum Present Value Formula

$$PV = \frac{FV}{(i+1)^n}$$

PV = present value

FV = future value

i = interest return per compounding period

n = number of compounding periods

Table 2.6

Now these formulas can be expanded to cover cases of multiple future payments and a short cut can be applied ***if*** the future payments are equal. If we apply the above PV formula to each payment, we can see how this works:

Present Value of Equal Payments

$$PV = PMT \times \frac{1-\left[\frac{1}{(1+i)^n}\right]}{i}$$

PMT = fixed periodic payment

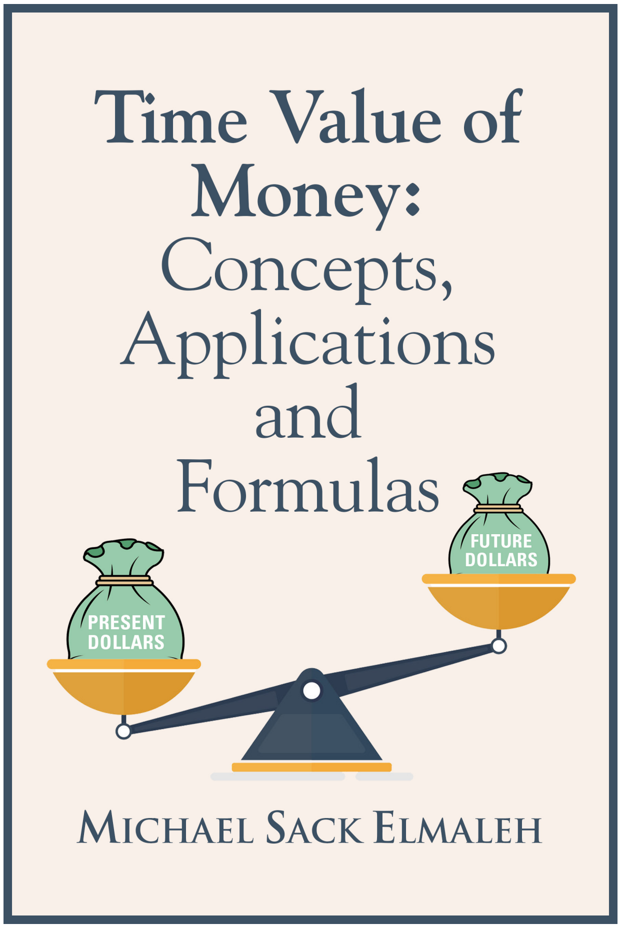
i = interest rate per period

n= number of future payment periods

Table 2.7

There is no need to memorize these formulas because even the simplest of calculators and every spreadsheet program has built in functions that will perform the computations. All you need to be careful of is to apply the functions in the right order. You also need to be aware of when to use these formulas…and when not to.

One thing you should be aware of is that these formulas are extremely sensitive to the choice of interest rates so perhaps the most important question to be answered is 'how do we choose an appropriate interest rate?" Before continuing with the mechanics and application of present value and compound interest formulas, in the next chapter I will discuss how interest rates are established.



A complete easy to understand guide to time value of money concepts, applications, and formulas. The book covers compound interest, opportunity costs, discount rates, interest rates, present and future values.

Time Value of Money: Concepts, Applications and Formulas

By Michael Sack Elmaleh