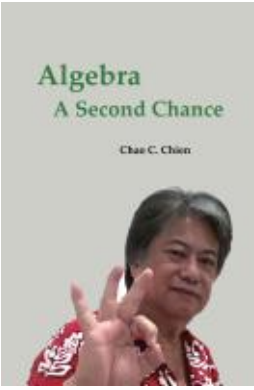
Algebra
A Second Chance

Chao C. Chien



Algebra is actually easier than many subjects of study. Its theories are in fact quite simple. Unfortunately many people have had bad experiences with algebra the first time through. This book tries to remedy that by showing how straightforward algebra is. It begins by explaining the basic concepts and building a solid foundation of understanding the basic learning elements, and then guides the reader through the various types of algebra equations step by step, systematically.

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Chao C. Chien

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Linear Equations

The simplest type of algebraic equations is the *linear equation*. It involves an unknown raised to a power no greater than 1, such as:

$$3x - 17 = 10$$

And it has a general form of:

$$ax + b = 0$$

In this equation, as we have already talked about, the unknown is denoted by *x*. *a*, and *b* are constants; that is, given values. *b* may or may not exist, but *a*, known as the *coefficient* of the unknown *x*, always does, because if *a* were zero, there would be no *x*, would it?

Linear equations are the easiest kind of algebraic equations to work out. In fact, it is so easy that when I set about to write this chapter all of a sudden I realized that there was hardly anything to write about because the solution is so simple. All you need to do is to remove all the clutter and leave the known bare on one side of the equation and you have it. So here we shall have it, for whatever that is worth mentioning.

As I said, to crack an equation, the goal is to isolate the unknown, to reduce the equation to the form of:

$$x = something$$

Therefore, all the efforts expended in solving a linear equation are directed toward collecting all the terms involving the unknown and placing them on one side of the equation and reducing the unknown's coefficient to 1, after all the rest have been removed. Taking our generalized linear equation "$ax + b = 0$" the approach is first to get rid of the term b on the left hand side of the equation, then removing the coefficient a (thus isolating x).

To remove b from the left hand side of the equation all we need to do is to subtract b from it. However, remember that if we do that and that alone we would have upset the equation. So, we must remember to do the same to the right side of the equation too: *whatever you do to one side of an equation you must immediately do the same to the other side; the ENTIRE side*. Bear in mind, as long as you do the same thing to both sides of an equation the equilibrium status of the equation remains true. So,

$$\begin{aligned} ax + b &= 0 \\ -b \quad &\quad -b \end{aligned}$$

We get:

$$ax = 0 - b \text{ (or, just } -b)$$

Next, we divide both sides of the equation by a (Why divide by a? Because a divided by a is 1):

$$\frac{ax}{a} = \frac{-b}{a}$$

Or,

$$x = -b / a$$

Suppose the right side of the equation is not 0. The procedure is still the same.

$$\begin{array}{l} ax + b = c \\ \quad\;\; - b \quad - b \end{array}$$

We get:

$$ax = c - b$$

Divide both sides of the equation by *a:*

$$\frac{ax}{a} = \frac{(c - b)}{a}$$

Note that I placed the expression on the right side of the equation in parentheses. This way I make it clear that the division by *a* is to apply to the ENTIRE right side and not just to *c* or *b,* which is a common error committed by beginners.

The answer is

$$x = (c - b) / a$$

Can we start by dividing both sides of the equation by *a* first so as to clean up *x* right away? Sure you can.

$$\frac{(ax + b)}{a} = \frac{c}{a}$$

That reduces to

$$\frac{ax}{a} + \frac{b}{a} = \frac{c}{a}$$

Or, because *a* divided by *a* gives 1,

$$x + \frac{b}{a} = \frac{c}{a}$$

Now subtract *b*/*a* from both sides of the equation.

$$x = \frac{c}{a} - \frac{b}{a}$$

Which is the same as

$$x = (c - b) / a$$

Either way we get there. You make the choice as to which precedence to take.

Let's see how the procedure works by applying it to real cases.

In cracking the following linear equation,

$$8x - 3 = 21$$

First we add 3 to both sides of the equation (aha, we add 3 instead of subtracting 3 because the original term in the equation on the left hand side is –3, which can be neutralized by a +3). That gives us:

$$8x - 3 + 3 = 21 + 3$$

After cleaning up,

$$8x = 24$$

Next, we divide both sides of the equation by 8. As a result, the left hand side becomes just *x* because 8 divided by 8 is 1. The right hand side of the equation is then 24 divided by 8, yielding 3:

$$\frac{8x}{8} = \frac{24}{8}$$

Giving us,

$$x = 3$$

Now check the results. (Hey, kids, if you are taking exams, check your results!) If we put the answer 3 back into the original equation we will have:

$$8(3) - 3 = 21$$
$$24 - 3 = 21$$

It checks out.

Well, that's linear equations for you, folks, simple, easy, and straight forward. So, if you have already grasped the materials presented above, you may choose to skip the rest of the chapter and move right along. Otherwise, let's try one more example.

$$7(x + 7) - 13 = 57$$

This equation looks complex only because of the set of parentheses. The parentheses here indicate that the 7 in front of it is to multiply the whole contents of the parentheses and not just the *x* next to it. We can remove the parentheses by virtue of the distributive law. Recall that:

$$a(b + c) = ab + ac$$

Thus, the left side of our example equation can be rewritten as:

$$7x + 7(7) - 13 = 57$$

Or

$$7x + 49 - 13 = 57$$

Next, leave the *7x* alone and deal with the 49 – 13 first. 49 – 13 is 36. So, we have:

$$7x + 36 = 57$$

Next, get rid of the 36 so that the left side of the equation is left with only an *x* term. We do this by subtracting 36 from the left side of the equation, but we remember to do the same thing to the right side of the equation too:

$$7x + 36 - 36 = 57 - 36$$

This leads to:

$$7x = 21$$

Finally, we isolate *x* by dividing both sides of the equation by 7, which yields:

$$x = 3$$

Plug this value of *x* back into the original equation and see if it checks out.

Let us try one more. How about this one?

$$5(2x + 3) = \frac{(79 - 4x)}{3}$$

Hey, no big deal. An equation is an equation.

We do not like fractions, so we multiply both sides of the equation by 3. That will give us

$$15(2x + 3) = (79 - 4x)$$

Next, remove the parentheses on the left by multiplying 15 into them, giving:

$$30x + 45 = 79 - 4x$$

We have *x* terms on both sides of the equation. We do not like that. So, let us get rid of one of them. One way would be to add 4x to both sides of the equation, which will give us

$$30x + 45 + 4x = 79 - 4x + 4x$$

After simplification (known as *gathering terms*),

$$34x + 45 = 79$$

We then subtract 45 from both sides of the equation, (why?)

$$34x + 45 - 45 = 79 - 45$$

That leaves us with

$$34x = 34$$

Finally, divide both sides of the equation by 34, we get

$$x = 1$$

Is the answer correct? You check it out.

Work out the following equation and see for yourself if you have gotten the feel for solving linear equations.

$$\frac{(x+3)}{2} = (x-3)$$

In summary, a linear equation can have at the most the following variations. First, the basic form is:

$$ax + b = c$$

If c is zero, we have

$$ax + b = 0$$

In this case, subtract b from both sides, then divide both sides by a, which will give us

$$x = \frac{-b}{a}$$

If b is zero, the equation becomes

$$ax = c$$

In that case, just divide both sides by *a*, which yields:

$$x = \frac{c}{a}$$

Linear Equation with Two Unknowns

The linear equations discussed in the last chapter were easy; well, relatively easy. In most cases, if the linear equations appeared to you as complex, that was not because they were complex per se, but rather, most likely, because the elements of the equations were scattered and appeared complicated, or that the arrangements of their terms appear to be complicated. The simplification of the equation, as we saw in the previous examples, such as removing parentheses, in fact involved not algebraic but basic arithmetic operations.

The type of linear equations from the last chapter involving only one unknown, x, is the simplest type of algebraic equations. It is like a Sudoku game with only 5 cells unfilled.

What if there are two unknowns, such as the following equation, how do we tackle it?

$$x + y = 17$$

Technically, such an equation cannot be solved because for every possible value of x you can always come up with a y that can satisfy the equation's constraints. For example, if we suppose x is 1, the equation is good if y is 16. 1 + 16 is 17. If you guess x to be 2, then y = 15 will make it work. Even if we make x = -1 we can see that y = 18. We say such an equation has no *unique solution*. There can be many solutions. In fact, there are limitless solutions.